Supplementary Information: Decentralization and the Gamble for Unity

September 5, 2016

Abstract

This Appendix contains supplementary information for the manuscript "Decentralization and the Gamble for Unity." Specifically, Section A proves that the unique equilibrium in strong regimes, i.e., the one characterized in Proposition 1, is strict. Section B provides Mathematica code for the comparative statics and graphs concerning decentralization.

A Strict Equilibria

Consider strategy profile σ. For $i = C, P$, let $B_i^\sigma(g)$ denote the set of i's best replies to profile σ at grievance g. Specifically, a_i resides in $B_i^\sigma(g)$ if and only if

$$a_i \in \arg\max_{a_i' \in A_i} U_i^\sigma(a_i'; g),$$

where $A_C = \{\varnothing, 0, 1\}$, $A_P = \{0, 1\}$ and U_i^σ is defined in the proof of Lemma 1. With this notation in hand, I introduce the notion of strict equilibria in the sense of Harsanyi (1973) and van Damme (1991).

Definition A.1 *An equilibrium σ is* strict *if $B_i^\sigma(g)$ is a singleton, that is, $|B_i^\sigma(g)| = 1$, for all grievance g and actors $i = C, P$.*

Thus, when an equilibrium is strict, every actor uses pure strategies. Furthermore, i's expected payoff from deviating at grievance g from equilibrium σ is strictly less than i's expected payoff from playing the action specified by σ at grievance g. Because of this, sufficiently small changes in the model's underlying payoff and transition parameters will not effect equilibrium strategies, as these strict inequalities will still be preserved. As discussed in van Damme (1991) and Doraszelski and Escobar (2010), this property ensures that strictness is one of strongest equilibrium refinements, implying other conditions such as stability, regularity, and essentialness.

With this definition in mind, the following proposition states the main result. Namely, that the unique equilibrium in strong regimes is strict. This is a non-trivial exercise due to the considerable theoretical restrictions placed on the state-action per-period utility functions, which substantially reduces the dimensionality of the parameter space. As in the proof of Proposition 1, I maintain the generic conditions that g^* is not an integer and that the Periphery strictly prefers to not mobilize at grievance $g^\dagger$.

Proposition A.1 *In strong regimes, i.e,* $\kappa_C < \pi_C^C$*, the unique equilibrium is strict.*

Proof. Let σ denote the unique equilibrium characterized in Proposition 1. I show that $|B_i^\sigma(g)| = 1$ for all i and all grievance g. Consider three cases.

Case 1: $g \leq g^\dagger$. Consider C's decision, where $\sigma_C(0; g) = 1$ and $V_C^\sigma(g) = U_C^\sigma(0; g) = \frac{\pi_C^C}{1-\delta} > 0$. If C grants independence, then $U_C^\sigma(\varnothing; g) = 0 < U_C^\sigma(0; g)$. In addition, if C represses, then

$$\begin{aligned} U_C^\sigma(1; g) &= \pi - \kappa_C + \delta V_C^\sigma(g+1) \\ &\leq \pi - \kappa_C + \delta \frac{\pi_C^C}{1-\delta} \\ &< \frac{\pi_C^C}{1-\delta} = U_C^\sigma(0; g), \end{aligned}$$

where the weak inequality follows because $\frac{\pi_C^C}{1-\delta} \geq V_C^\sigma(g+1)$ (as in Lemma 1) and the strict inequality follows because $\kappa_C > 0$. Thus, $U_C^\sigma(0; g) > U_C^\sigma(\varnothing; g)$ and $U_C^\sigma(0; g) > U_C^\sigma(1; g)$ imply $B_C^\sigma(g) = \{0\}$, as required.

Consider P's decision, where $V_P^\sigma(g) = U_P^\sigma(0; g) = \frac{\pi_P^C}{1-\delta}$. It suffices to show $U_P^\sigma(0; g) > U_P^\sigma(1; g)$. As described in Lemma 1, this inequality holds if and essentially only if

$$\kappa_P > F(g) \left[\frac{\pi_P^P}{1-\delta} - \pi_P^C - \delta V_P^\sigma(\max\{g-1, 0\}) \right].$$

Because $g \leq g^\dagger$ implies $V_P^\sigma(\max\{g-1, 0\}) = \frac{\pi_P^C}{1-\delta}$, then the inequality is equivalent to

$$\begin{aligned} \kappa_P &> F(g) \left[\frac{\pi_P^P}{1-\delta} - \pi_P^C - \delta \frac{\pi_P^C}{1-\delta} \right] \\ &> F(g) \frac{\pi_P^P - \pi_P^C}{1-\delta}. \end{aligned}$$

Then $U_P^\sigma(0; g) > U_P^\sigma(1; g)$ is equivalent to $g \leq g^\dagger$, which holds by assumption.

Case 2: $g > g^*$. Consider C's decision, where $\sigma_C(1; g) = 1$ and $V_C^\sigma(g) = U_C^\sigma(1; g) = \frac{\pi_C^C - \kappa_C}{1-\delta} > 0$. As in the previous case, $\{\varnothing\} \not\subset B_C^\sigma(g)$, so it suffices to show that

$U_C^\sigma(1;g) > U_C^\sigma(0;g)$. If C chooses $r=1$, then it's payoff is

$$U_C^\sigma(1;g) = -F(g)\psi + (1-F(g))\left(\pi_C^C + \delta V_C^\sigma(g-1)\right).$$

If $g-1 < g^*$, then we have

$$\begin{aligned} U_C^\sigma(1;g) &= \tilde{V}_C(g) \\ &< \frac{\pi_C^C - \kappa_C}{1-\delta} \\ &= U_C^\sigma(1;g) \end{aligned}$$

by the construction of g^*. If $g-1 > g^*$, then we have

$$\begin{aligned} U_C^\sigma(1;g) > U_C^\sigma(0;g) &\iff \frac{\pi_C^C - \kappa_C}{1-\delta} > -F(g)\psi + (1-F(g))\left(\pi_C^C + \delta\frac{\pi_C^C - \kappa_C}{1-\delta}\right) \\ &\iff \kappa_C < \frac{F(g)(\pi_C^C + \psi(1-\delta))}{1-(1-F(g))\delta}. \end{aligned}$$

This last strict inequality must hold, however. If not, then $\tilde{V}_C(g) > U_C^\sigma(1;g)$ by the logic Lemma 4. Then $\tilde{V}_C(g) > U_C^\sigma(1;g) = V_C^\sigma(g)$ contradicts Lemma 3. Thus, $\{0\} \not\subset B_C^\sigma(g)$, so $B_C^\sigma(g) = \{1\}$, as required.

Next, Consider P's decision, where $\sigma_P(g) = 1$. Similar to Case 1, it suffices to show $U_P^\sigma(1;g) > U_P^\sigma(0;g)$, and this inequality holds if and only if

$$\kappa_P < F(g)\left[\frac{\pi_P^P}{1-\delta} - \pi_P^C - \delta V_P^\sigma(g-1)\right]. \tag{1}$$

If $g-1 > g^*$, then $V_P^\sigma(g-1) = \frac{\pi_C^C}{1-\delta}$. Substituting this value into the inequality in Eq. 1 reveals that $U_P^\sigma(1;g) > U_P^\sigma(0;g)$ if and only if

$$\kappa_P < F(g)\left[\frac{\pi_P^P - \pi_C^P}{1-\delta}\right].$$

But this last inequality must be true because $g > g^*$ implies $g > g^\dagger$. If $g-1 < g^*$, then $V_P^\sigma(g-1) \leq \frac{F(g)\bar{V}_P + (1-F(g))\pi_P^C - \kappa_P}{1-(1-F(g))\delta}$ by the logic in Lemma 5. Combining this inequality with the one in Eq. 1 reveals that $U_P^\sigma(0;g) > U_P^\sigma(1;g)$ if the following inequality holds:

$$\kappa_P < F(g)\left[\frac{\pi_P^P}{1-\delta} - \pi_C^P - \delta\frac{F(g)\bar{V}_P + (1-F(g))\pi_P^C - \kappa_P}{1-(1-F(g))\delta}\right].$$

Solving for κ_P, $U_P^\sigma(0;g) > U_P^\sigma(1;g)$ if the following holds:

$$\kappa_P < F(g)\left[\frac{\pi_P^P - \pi_C^P}{1-\delta}\right].$$

But again, this last inequality must hold because $g > g^*$ implies $g > g^\dagger$.

Case 3: $\max G_1 < g < g^*$. Consider C's decision, where $\sigma_C(0;g) = 1$ and $V_C^\sigma(g) = \tilde{V}_C(g)$. As in the previous two cases, $\{\varnothing\} \not\subset B_C^\sigma(g)$. Thus, it suffices to show that $U_C^\sigma(1;g) < U_C^\sigma(0;g) = \tilde{V}_C(g)$, where $U_C^\sigma(1;g)$ takes the form:

$$U_C^\sigma(1;g) = \pi_C^C - \kappa_C + \delta V_C^\sigma(g+1).$$

If $g+1 > g^*$, then $U_C^\sigma(1;g) = \frac{\pi_C^C - \kappa_C}{1-\delta}$. Furthermore, $g < g^*$ implies $\tilde{V}_C(g) < \frac{\pi_C^C - \kappa_C}{1-\delta}$. Thus, $U_C^\sigma(1;g) < U_C^\sigma(0;g)$. If $g+1 < g^*$, then

$$\begin{aligned} U_C^\sigma(1;g) &= \pi_C^C - \kappa_C + \delta\tilde{V}_C(g+1) \\ &< \pi_C^C - \kappa_C + \delta\tilde{V}_C(g-1) \\ &\leq -F(g)\psi + (1-F(g))\left(\pi_C^C + \delta\tilde{V}_C(g-1)\right) \\ &= \tilde{V}_C(g) = U_C^\sigma(0;g). \end{aligned}$$

Here, the first line follows because $g+1 < g^*$ and $g+1 > g^\dagger$. The second line follows because $g > g^\dagger$ and Lemma 2(2) imply $\tilde{V}_C$ is strictly decreasing. The third line follows because $g < g^*$. The fourth line follows by construction of $\tilde{V}_C$ and the equilibrium σ. Hence, $U_C^\sigma(1;g) < U_C^\sigma(0;g)$, and $B_C^\sigma(g) = \{0\}$.

Finally, an identical argument as in Case 2 shows that $B_P^\sigma(g) = \{1\}$. □

B Mathematica Code for Decentralization Graphs

Listing 1: Decentralization Comparative Statics

```
(* PRELIMS *)
ClearAll["Global`*"]
Attributes[\[Pi]] = {};
$RecursionLimit = 5000; (* Becareful here *)

(* PARAMETERS *)
\[Pi] = 100;
\[Kappa]C = 50;
\[Kappa]R = 300;
(*\[Psi] = 100;*)
\[Delta] = .95;
F[g_] := F[g] = (1 - 1/(0.01 g + 1));
```

```
(* PRELIMINARY FUNCTIONS *)
(* compute \bar{p} *)

bp = Limit[F[g], g -> Infinity];

(* Compute max G_1 *)

maxG1[d_] :=
  maxG1[d] =
   If[\[Kappa]R - bp (\[Pi] - d)/(1 - \[Delta]) >= 0, Infinity,
     First[
      Flatten[Position[\[Kappa]R -
            F[#] (\[Pi] - d)/(1 - \[Delta]) & /@
          Range[1, 5000], _?(# < 0 &)]]]] - 1;

(* \tilde{V}_C(g), recursive *)

tVC[g_, d_, \[Psi]_] :=
  tVC[g, d, \[Psi]] =
   If[g <= maxG1[
       d], (\[Pi] -
        d)/(1 - \[Delta]), -F[g] \[Psi] + (1 - F[g]) (\[Pi] -
         d + \[Delta] tVC[g - 1, d, \[Psi]])];

(* Limit of \tilde{V}_C(g) under assumption 1 *)

limitV[d_, \[Psi]_] := ((1 - bp) (\[Pi] - d) -
      bp \[Psi])/(1 - \[Delta] (1 - bp));

(* Expected utility of long-term repression *)

Vrep[d_] := (\[Pi] - d - \[Kappa]C)/(1 - \[Delta]);

(* Computes g^* from Propositions 1 and 2 *)

gStar[d_, \[Psi]_] :=
  First[Flatten[
      Position[
       tVC[#, d, \[Psi]] -
           Max[(\[Pi] - d - \[Kappa]C)/(1 - \[Delta]), 0] & /@
         Range[0, 1000], _?(# < 0 &)]]] - .5;

(* Expected utility of the Center, strong regimes *)

EUC[g_, d_, \[Psi]_] := Max[tVC[g, d, \[Psi]], Vrep[d], 0];

(* PR national unity, strong regimes *)

PRunity[g_, d_, \[Psi]_] :=
 Module[{temp}, temp = EUC[g, d, \[Psi]];
```

```
  If[temp == Vrep[d], 1,
   If[temp == 0, 0, Product[1 - F[j], {j, g, maxG1[d], -1}]]]]

(* PR national unity, secession, strong regimes *)

PRsecede[g_, d_, \[Psi]_] :=
 Module[{temp}, temp = EUC[g, d, \[Psi]];
  If[temp == Vrep[d], 0,
   If[temp == 0, 0, 1 - Product[1 - F[j], {j, g, maxG1[d], -1}]]]]

(* compute optimal decentralization level, requires assumption from \
prop 6*)
Findd[g_] :=
 If[ g <= maxG1[0], 0,
  d /. FindRoot[\[Kappa]R - F[g] (\[Pi] - d)/(1 - \[Delta]), {d,
      0, \[Pi]}, Method -> "Brent"][[1]]]
FinddAll[g1_] :=
 FindAll[g1] =
  Map[Findd, Range[Floor[maxG1[0]], Max[Floor[maxG1[0]], g1]]]
 dStar[g1_, \[Psi]_] :=
 FinddAll[g1][[Ordering[
     MapThread[
      EUC, {PadLeft[{g1}, Length[FinddAll[g1]], g1], FinddAll[g1],
        PadLeft[{\[Psi]}, Length[FinddAll[g1]], \[Psi]]}], -1][[1]]]]


(* PLOTS *)
(* g^* plot *)
DiscretePlot[{gStar[d, \[Pi]/5] +
   3(*added for effect*), gStar[d, \[Pi]]}, {d, 0, 100, 1},
 Frame -> {True, True, False, False}, Axes -> False,
 FrameLabel -> {" ", " "}, RotateLabel -> False,
 Filling -> None, PlotMarkers -> {Automatic, 10},
 PlotLegends ->
  PointLegend[ {Style[" ", Bold, FontSize -> 20],
    Style[" ", Bold, FontSize -> 20]}, LegendMarkers -> Automatic],
 LabelStyle -> Directive[FontSize -> 48],
 Joined -> Automatic,
 FrameTicks -> {{{\[Pi] - \[Kappa]C, " ", {62/100, 0},
     Directive[Dashed]}, {50,
     " ", {0, 0}}, {\[Pi] - \[Kappa]R (1 - \[Delta])/bp,
     " ", {62/100, 0}, Directive[Dashed]}}, {}}
 ]

(* PR national unity plot plot *)
DiscretePlot[{PRunity[30,
   d, \[Pi]/5]}, {d, 0, \[Pi] - \[Kappa]C, .05},
       Frame -> {True, True, False, False}, Axes -> False,
 FrameLabel -> {"     ", "               "}, RotateLabel -> False,
 Filling -> None, PlotMarkers -> {Automatic, 10},
```

```
 LabelStyle -> Directive[FontSize -> 24],
 FrameTicks -> { {{\[Kappa]C, "␣", {61/100, 0},
     Dashed}, {\[Pi] - (1 - \[Delta]) \[Kappa]R/F[30],
     "␣", {61/100, 0}, Dashed}, {20, "␣", {0, 0}, Dashed }}, {{0.25,
     "␣"}, {0.5, "␣"}, {0.75, "␣"}, {1.0, "␣"}}}
 ]

(* optimal decentralization plot *)
DiscretePlot[
 dStar[g1, \[Pi]/5], {g1, 0, 50, 1}, Filling -> None,
 Frame -> {True, True, False, False}, Axes -> False,
 FrameLabel -> {"␣␣", "␣␣␣␣␣"}, RotateLabel -> False,
 PlotMarkers -> {Automatic, 7},
 LabelStyle -> Directive[FontSize -> 18],
 PlotLabel -> Style["␣␣␣", FontSize -> 20],
 FrameTicks -> {{{17, "␣␣"}, {42.5, "␣␣"}, {70, "␣"}}, {{10,
     "␣␣"}, {20, "␣␣"}, {30, "␣␣"}, {40, "␣␣"}, {50, "␣␣"}}}
 ]


(* EUC plot for endognenous decentralization *)

p1 = DiscretePlot[{EUC[10, d, \[Pi]/5]}, {d, 0, \[Kappa]C, 0.1},
   Filling -> None, PlotRange -> {0, \[Pi]/(1 - \[Delta])},
   Frame -> {True, True, False, False}, Axes -> False,
   FrameLabel -> {d,
     "\!\(\*SubsuperscriptBox[\(V\),␣\(C\),␣\(\[Sigma]\)]\)(10)"},
   RotateLabel -> False,
   PlotMarkers -> {Automatic, 8},
   LabelStyle -> Directive[FontSize -> 20],
   PlotLabel ->
    Style["\!\(\*SuperscriptBox[\(g\),␣\(1\)]\)=10", FontSize -> 20],
   FrameTicks -> {{{\[Pi] - \[Kappa]C,
       "\[Pi]-\!\(\*SubscriptBox[\(\[Kappa]\),␣\(C\)]\)", {60/100, 0},
        Dashed}, {50, "␣", {0, 0}}, {\[Pi],
       "\[Pi]"}}, {{\[Pi]/(1 - \[Delta]),
       "\!\(\*FractionBox[\(\[Pi]\),␣\(1␣-␣\[Delta]\)]\)"}}}
   ];
p2 = DiscretePlot[{EUC[20, d, \[Pi]/5]}, {d, 0, \[Kappa]C, 0.1},
   Filling -> None, PlotRange -> {0, \[Pi]/(1 - \[Delta])},
   Frame -> {True, True, False, False}, Axes -> False,
   FrameLabel -> {d,
     "\!\(\*SubsuperscriptBox[\(V\),␣\(C\),␣\(\[Sigma]\)]\)(20)"},
   RotateLabel -> False,
   PlotMarkers -> {Automatic, 8},
   LabelStyle -> Directive[FontSize -> 20],
   PlotLabel ->
    Style["\!\(\*SuperscriptBox[\(g\),␣\(1\)]\)=20", FontSize -> 20],
   FrameTicks -> {{{\[Pi] - \[Kappa]C,
       "\[Pi]-\!\(\*SubscriptBox[\(\[Kappa]\),␣\(C\)]\)", {60/100, 0},
```

```
        Dashed}, {\[Pi] - \[Kappa]R (1 - \[Delta])/F[20],
       "\[Pi] - \!\(\*FractionBox[\(\*SubscriptBox[\(\[Kappa]\), \
\(R\)] \((1 - \[Delta])\)\), \(F \((20)\)\)]\)", {60/100, 0},
       Dashed}, {\[Pi], "\[Pi]"}}, {{\[Pi]/(1 - \[Delta]),
       "\!\(\*FractionBox[\(\[Pi]\), \(1 - \[Delta]\)]\)"}}}
   ];
p3 = DiscretePlot[{EUC[30, d, \[Pi]/5]}, {d, 0, \[Kappa]C, 0.1},
   Filling -> None, PlotRange -> {0, \[Pi]/(1 - \[Delta])},
   Frame -> {True, True, False, False}, Axes -> False,
   FrameLabel -> {d,
     "\!\(\*SubsuperscriptBox[\(V\), \(C\), \(\[Sigma]\)]\)(30)"},
   RotateLabel -> False,
   PlotMarkers -> {Automatic, 8},
   LabelStyle -> Directive[FontSize -> 18],
   PlotLabel ->
    Style["\!\(\*SuperscriptBox[\(g\), \(1\)]\)=30", FontSize -> 20],
   FrameTicks -> {{{\[Pi] - \[Kappa]C,
       "\[Pi]-\!\(\*SubscriptBox[\(\[Kappa]\), \(C\)]\)", {60/100, 0},
        Dashed}, {\[Pi] - \[Kappa]R (1 - \[Delta])/F[30],
       "\[Pi] - \!\(\*FractionBox[\(\*SubscriptBox[\(\[Kappa]\), \
\(R\)] \((1 - \[Delta])\)\), \(F \((30)\)\)]\)", {60/100, 0},
       Dashed}, {\[Pi], "\[Pi]"}}, {{\[Pi]/(1 - \[Delta]),
       "\!\(\*FractionBox[\(\[Pi]\), \(1 - \[Delta]\)]\)"}}}
   ];
p4 = DiscretePlot[{EUC[50, d, \[Pi]/5]}, {d, 0, \[Kappa]C, 0.1},
   Filling -> None, PlotRange -> {0, \[Pi]/(1 - \[Delta])},
   Frame -> {True, True, False, False}, Axes -> False,
   FrameLabel -> {d,
     "\!\(\*SubsuperscriptBox[\(V\), \(C\), \(\[Sigma]\)]\)(50)"},
   RotateLabel -> False,
   PlotMarkers -> {Automatic, 8},
   LabelStyle -> Directive[FontSize -> 20],
   PlotLabel ->
    Style["\!\(\*SuperscriptBox[\(g\), \(1\)]\)=50", FontSize -> 20],
   FrameTicks -> {{{\[Pi] - \[Kappa]C,
       "\[Pi]-\!\(\*SubscriptBox[\(\[Kappa]\), \(C\)]\)", {60/100, 0},
        Dashed}, {50, " ", {0, 0}}, {\[Pi],
       "\[Pi]"}}, {{\[Pi]/(1 - \[Delta]),
       "\!\(\*FractionBox[\(\[Pi]\), \(1 - \[Delta]\)]\)"}, {(\[Pi]/
          2)/(1 - \[Delta]),
       " "}, {(\[Pi] - \[Kappa]C)/(1 - \[Delta]),
       "\!\(\*FractionBox[\(\[Pi] - \[Kappa]C\), \(1 - \
\[Delta]\)]\)"}}}
   ];
GraphicsGrid[{{p1, p2}, {p3, p4}}]
```

References

Doraszelski, Ulrich and Juan F. Escobar. 2010. "A Theory of Regular Markov Perfect Equilibria in Dynamic Stochastic Games: Genericity, Stability, and Purification." *Theoretical Economics* 5(3):369–402.

Harsanyi, John C. 1973. "Oddness of the Number of Equilibrium Points: A New Proof." *International Journal of Game Theory* 2(1):235–250.

van Damme, Eric. 1991. *Stability and Perfection of Nash Equilibria.* Second ed. New York: Springer.